Feature Article:
MH370 Burst Frequency Offset Analysis and Implications on Descent Rate at End of Flight

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INTRODUCTION

On 7 March 2014 at 16:41:43Z,[1] Malaysian Airlines flight MH370 departed Kuala Lumpur (KL) International Airport bound for Beijing. Less than an hour later, following the last recorded radio transmission from MH370 at 17:19:30Z, the plane's secondary radar transponder went offline. As evidenced by Malaysian military radar, the plane (registration number 9M-MRO) then veered off course unexpectedly, backtracked across the Malaysian Peninsula, and was tracked heading northwest from Penang through the Malacca Straits. After disappearing from radar at 18:22:12Z, it reestablished a satellite communications (SATCOM) link with the Inmarsat satellite I-3F1 at 18:25:27Z. By analyzing a series of automated messages exchanged via that satellite between the plane and an Inmarsat ground station in Perth, Australia, it was determined that the plane continued to fly for 6 h before finally ceasing message exchange with the ground station at 00:19:37Z on 8 March 2014. This article discusses specifically the analysis of burst frequency offset (BFO) metadata from the SATCOM messages. It is shown that the BFOs corresponding to the last two SATCOM messages from the plane at 00:19:29Z and 00:19:37Z suggest that flight MH370 was rapidly descending and accelerating downward when message exchange with the ground station ceased.

An initial analysis by Inmarsat of the SATCOM metadata for MH370 in the last 6 h of flight suggested that MH370 had flown into the Southern Indian Ocean before SATCOM was lost (see [1] for further details). As summarized in [2], an intensive aerial and surface search was undertaken in the Southern Indian Ocean by an international search team during March and April 2014, with no MH370-related debris found. On 28 April 2014, the aerial search concluded, and the search transitioned to an underwater phase [2]. The Australian Transport Safety Bureau (ATSB) took responsibility for the definition of the underwater search zone. It convened an international flight path prediction working group, bringing together

er experts in SATCOM, statistical data processing, and aviation, to estimate the most likely final location of flight MH370. The group consisted of representatives from the Australian Defence Science and Technology (DST) Group and the other organizations listed in the Acknowledgments section of this article.

New methods of analyzing the Inmarsat data were developed by the group, resulting in the release of reports concerning the likely final location of flight MH370 from the ATSB in August 2014 [3], October 2014 [4], and December 2015 [5]. Inmarsat also published an article regarding its contribution to the flight path reconstruction effort [1]. The DST Group contribution that assisted in the definition of an extended priority search area in December 2015 [5] has been detailed in [2]. This demonstrated how Bayesian analysis was used to identify a high-probability region of where the plane was believed to be at the time of last SATCOM transmission (00:19:37Z, 8 March 2014). The DST Group Bayesian method used a prior probability distribution defined by the Malaysian military radar, a likelihood function describing the relationship between SATCOM measurements and aircraft position and velocity during the flight, and a model of the aircraft dynamics.

This article does not cover the Bayesian method used in defining the underwater search area. For details on that method, refer to [2]. Instead, this article focuses on the following:

1. A review of the statistical analysis of BFOs for several previous flights of 9M-MRO

2. Examination of the effects of the plane's vertical velocity on the BFO

3. An analysis of the effect of the track angle of the plane on the BFO toward the end of flight

4. The establishment of a BFO trend throughout the last 6 h of flight so as to determine an expected BFO at end of flight

5. An analysis of the behavior of the frequency oscillator in the plane's satellite data unit (SDU) after power outage events, such as those believed to have occurred twice during flight MH370, and how this affects the BFO

6. An analysis that shows that the final two BFOs are consistent with MH370 being in a rapid descent and accelerating downward

This article first presents a timeline of key events during the MH370 accident flight. Then a review of the SATCOM model is provided, along with a review of the BFO statistics. This serves as a

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[1] All times given in this article are in UTC or Zulu time, denoted by Z.
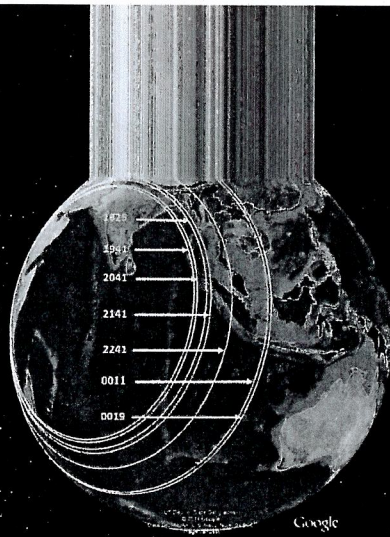
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summary of work previously presented in [2]. The next two sections describe the effects of aircraft position and velocity on the BFO and the settling behavior of the SDU's frequency oscillator for 9M-MRO after power-up. This is important because the SDU is believed to have undergone a power outage between 00:11Z and 00:19Z on 8 Mar 2014, immediately preceding the last two SATCOM transmissions from MH370. The effect of oscillator warm-up on the BFO after power-up can be used in bounding the descent rates for MH370 at 00:19:29Z and 00:19:37Z. Finally, the last section is an analysis of the descent rate of MH370 based on the last two BFO values. This analysis derives lower and upper bounds on the descent rate at 00:19:29Z and 00:19:37Z.

TIMELINE OF EVENTS DURING THE ACCIDENT FLIGHT

A comprehensive description of the events occurring during the MH370 accident flight is provided in [7]. A summary of salient events to this article is provided in Table 1.

REVIEW OF SATCOM MODEL

The accident aircraft was fitted with a SATCOM terminal that used the Inmarsat Classic Aero system [1], which uses geosynchronous satellites to relay messages between aircraft and ground stations.

During the flight, messages were passed between the aircraft and a ground receiving station located in Perth, Australia, via the Inmarsat-3F1 satellite. Figure 1 illustrates the SATCOM system in use during the flight. The aircraft is called the Aircraft Earth Station (AES), and the ground receiving unit is called the Ground Earth Station (GES). Inmarsat-3F1 is a satellite in geosynchronous orbit at 64.5° E, and it was used exclusively for the duration of the flight.

An AES is equipped with an SDU comprising a satellite modem with auxiliary hardware and software. Transmission of data
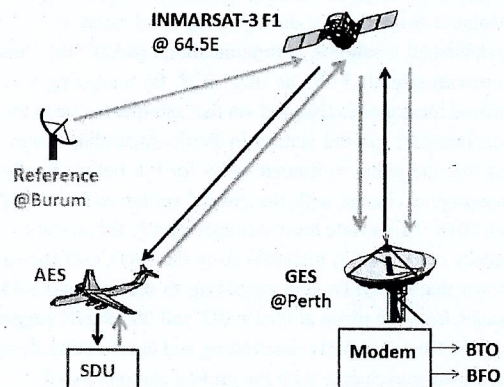

Figure 1.
System model of the satellite communication system [2].

Table 1.

Timeline of Events During MH370

Date and Time Stamps	Description	Comments
7 Mar. 2014 16:42Z	MH370 departs Kuala Lumpa Airport	Normal takeoff
Last ACARS transmission	7 Mar. 2014 17:07Z	Everything normal, on path to Beijing
Loss of secondary radar	7 Mar. 2014 17:21:13Z	Onboard transponder goes inactive; last civilian radar
7 Mar. 2014 18:22:12Z	Last Malaysian military radar contact	MH370 shown tracking NW through Malacca Straits
7 Mar. 2014 18:25Z–18:28Z	SDU log-on sequence	If flying level, MH370 still tracking NW
7 Mar. 2014 18:39–18:41Z	Unanswered ground-to-air phone call	If flying level, MH370 tracking southward
7 Mar. 2014 19:41Z to 8 Mar. 2014 00:11Z	SATCOM metadata approx. hourly from MH370	[1], [2] suggest flight into Southern Indian Ocean
8 Mar. 2014 00:19:29Z–00:19:37Z	Partial SDU log-on sequence	Implies rapid descent

over the satellite is via bursts, which are scheduled to arrive at the GES at a specified time and a given frequency. As explained in [1] and [2], communications from multiple users are coordinated by the allocation of different time and frequency slots to each user. This is done without knowledge of individual AES locations or precise knowledge of the satellite location. Therefore, messages from a given AES might not arrive at the GES at exactly the expected time and generally would arrive slightly later. The difference between the expected time of arrival (based on a nominal assumed position for the satellite and the AES) and the actual time of arrival is called the burst timing offset (BTO). The BTO is a measure of how far the aircraft is from the subsatellite position.[2]

The relative velocity between the satellite and the AES, as well as between the satellite and the GES, leads to a Doppler frequency offset on the signals received at the GES. Coupled with small frequency offsets inherent in the reference frequency oscillators in the AES, satellite, and GES, this results in a net difference between the expected and the actual frequencies of the signal presented to the modem in the GES for a given user. Frequency compensations applied onboard the aircraft (aircraft-induced Doppler precompensation) and at the ground station (enhanced automatic frequency correction, which uses the reference signal transmitted from a reference station in Burum, the Netherlands), [1] and [2] serve to reduce the possible difference between the expected and the actual frequencies of the messages received from the aircraft. The residual difference between the expected frequency of each communications burst and the actual received frequency is called the BFO.

REVIEW OF BFO STATISTICS

Based on 20 previous flights of 9M-MRO in the week leading up to the accident flight (see [2] for further details), a histogram was produced for the difference between the predicted BFO (based on known details of the plane and the satellite's position and velocity) and the measured BFO (based on Inmarsat ground-station logs). This difference (i.e., predicted minus measured) is called the BFO error. The histogram of the BFO error is shown in Figure 2, along with a Gaussian distribution fit line. It can be seen that the distribution is somewhat Gaussian. The standard deviation of the BFO error was found in [2] to be 4.3 Hz. While it is reasonable to apply bounds on the possible BFO error based on ±3 standard deviations, as was done for the approximate analysis described in [6], for the purpose of the descent analysis presented later in this article, it is assumed the BFO error is strictly bounded on the larger interval [−28, +18] Hz, which corresponds to the bounds of all 2,501 observed valid[3] in-flight BFO error values available from the preceding 20 flights of 9M-MRO.
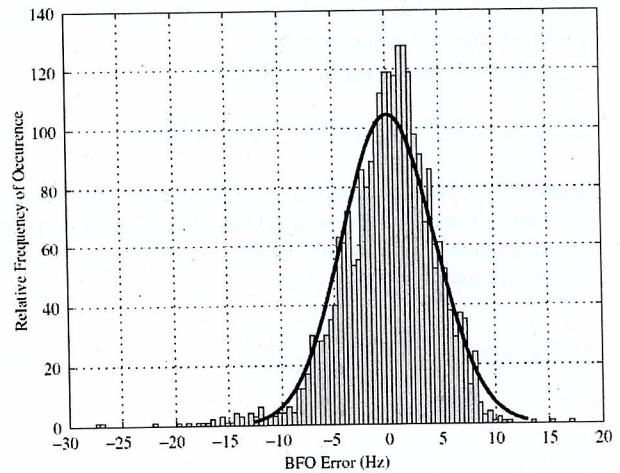
EFFECTS OF AIRCRAFT POSITION AND VELOCITY ON THE BFO

MATHEMATICAL DESCRIPTION OF THE BFO

In [2], the BFO is defined mathematically at time step k as the sum of a noiseless component h_k^{BFO} and a scalar w_k^{BFO} that represents the

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[2] The subsatellite position is the point on Earth directly below the satellite.
[3] One outlier was removed, as explained later in this article.

Figure 2.
Histogram of BFO errors for 20 flights of 9M-MRO before MH370 (reproduced from Figure 5.5 of [2]). The mean and standard deviation for the distribution are 0.18 and 4.3 Hz, respectively [2].

BFO noise. The noiseless component of the BFO is defined in [2] as the sum

$$h_k^{\text{BFO}}\left(\mathbf{x}_k, \mathbf{s}_k\right) = \Delta F_k^{\text{up}}\left(\mathbf{x}_k, \mathbf{s}_k\right) + \Delta F_k^{\text{down}}\left(\mathbf{s}_k\right)$$
$$+ \delta f_k^{\text{comp}}\left(\mathbf{x}_k\right) + \delta f_k^{\text{sat}}\left(\mathbf{s}_k\right) \qquad (1)$$
$$+ \delta f_k^{\text{AFC}}\left(\mathbf{s}_k\right) + \delta f_k^{\text{bias}}\left(\mathbf{x}_k, \mathbf{s}_k\right),$$

where

▶ $\mathbf{x}_k$ denotes the state vector of the aircraft;

▶ $\mathbf{s}_k$ denotes the state vector of the satellite;

▶ $\Delta F_k^{\text{up}}\left(\mathbf{x}_k, \mathbf{s}_k\right)$ is the uplink (aircraft to satellite) Doppler shift;

▶ $\Delta F_k^{\text{down}}\left(\mathbf{s}_k\right)$ is the downlink (satellite to ground station) Doppler shift;

▶ $\delta f_k^{\text{comp}}\left(\mathbf{x}_k\right)$ is the frequency compensation applied by the aircraft;

▶ $\delta f_k^{\text{sat}}\left(\mathbf{s}_k\right)$ is the variation in satellite translation frequency;

▶ $\delta f_k^{\text{AFC}}\left(\mathbf{s}_k\right)$ is the frequency compensation applied by the ground-station receive chain; and

▶ $\delta f_k^{\text{bias}}\left(\mathbf{x}_k, \mathbf{s}_k\right)$ is a slowly varying bias because of errors in the aircraft and satellite oscillators and processing in the SDU.

By treating the bias $\delta f_k^{\text{bias}}\left(\mathbf{x}_k, \mathbf{s}_k\right)$ as a constant determined at the source tarmac for any particular flight, as was done in [2] for MH370, any small time-varying component of the bias during a particular flight can be considered part of the BFO noise (this was done when compiling the results used to obtain the BFO error histogram shown in Figure 2). Details regarding the terms $\delta f_k^{\text{sat}}\left(\mathbf{s}_k\right)$ and $\delta f_k^{\text{AFC}}\left(\mathbf{s}_k\right)$ are provided in [1]. Tabulated values of the sum of these two terms were provided by Inmarsat to the MH370 Flight Path Reconstruction Group to use in estimating the likely trajectory flown. These two terms depend on the satellite state $\mathbf{s}_k$ only, not on the aircraft state $\mathbf{x}_k$. Moreover, the downlink Doppler $\Delta F_k^{\text{down}}\left(\mathbf{s}_k\right)$ does not depend on the location or velocity of the aircraft and can

be calculated given the known frequency of the downlink and the known satellite state at any given time.

Equation (1) can be then be simplified to

$$h_k^{\text{BFO}}\left(\mathbf{x}_k,\mathbf{s}_k\right)=\Delta F_k^{\text{up}}\left(\mathbf{x}_k,\mathbf{s}_k\right)+\delta f_k^{\text{comp}}\left(\mathbf{x}_k\right)+\delta f_k^{\text{det}}\left(\mathbf{s}_k\right), \qquad (2)$$

where $\delta f_k^{\text{det}}\left(\mathbf{s}_k\right)$ is effectively a known deterministic value for any time step k. The other terms in (2) couple the aircraft state $\mathbf{x}_k$ by way of the aircraft position and velocity to the BFO as per the following equations adapted from [2][4]:

$$\Delta F_k^{\text{up}}\left(\mathbf{x}_k,\mathbf{s}_k\right)=\frac{F^{\text{up}}}{c}\frac{\left(\mathbf{v}_s-\mathbf{v}_x\right)^{\text{T}}\left(\mathbf{p}_x-\mathbf{p}_s\right)}{\left|\mathbf{p}_x-\mathbf{p}_s\right|}, \qquad (3)$$

$$\delta f_k^{\text{comp}}\left(\mathbf{x}_k\right)=\frac{F^{\text{up}}}{c}\frac{\left(\hat{\mathbf{v}}_x\right)^{\text{T}}\left(\hat{\mathbf{p}}_x-\hat{\mathbf{p}}_s\right)}{\left|\hat{\mathbf{p}}_x-\hat{\mathbf{p}}_s\right|}, \qquad (4)$$

where the dependence on the time step k on the right-hand side of the equations has been removed for simplicity of notation. In (3) and (4),

- ▶ $\left|\cdot\right|$ is the three-dimensional Cartesian distance;
- ▶ F^{up} is the uplink carrier frequency;
- ▶ c is the speed of light;
- ▶ $\mathbf{v}_s$ is the velocity vector of the satellite;
- ▶ $\mathbf{v}_x$ is the velocity vector of the plane;
- ▶ $\mathbf{p}_s$ is the position vector of the satellite;
- ▶ $\mathbf{p}_x$ is the position vector of the plane;
- ▶ $\hat{\mathbf{v}}_x$ is the SDU's estimate of the plane's velocity vector, which is obtained using the plane's track angle and ground speed, while assuming the vertical speed is zero;
- ▶ $\hat{\mathbf{p}}_s$ is the SDU's estimate of the position vector of the satellite, which assumes the satellite is at its nominal oribital slot of 0° N and 64.5° E; and
- ▶ $\hat{\mathbf{p}}_x$ is the SDU's estimate of the position vector of the plane, which is obtained using the plane's latitude and longitude while assuming the plane is at sea level.

EFFECT OF UNCOMPENSATED VERTICAL VELOCITY

The vertical speed of the plane is not used in the SDU Doppler compensation. As such, there is a direct contribution of Doppler because of the proportion of the vertical velocity vector projected onto the radial direction from the aircraft to the satellite. It is straightforward to understand that if the plane was directly below the satellite, the vertical velocity vector would be fully toward or from the satellite if the plane was climbing or descending. The direct contribution of the Doppler to the BFO in that case would be governed by the following standard Doppler equation:

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[4] The sign convention used in [2] is opposite that used in (3).
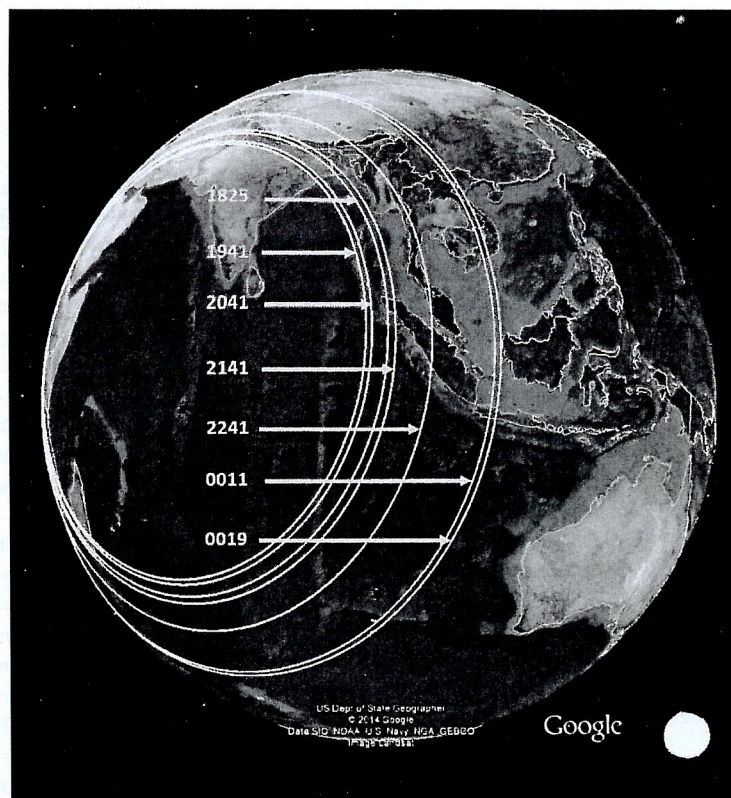
Figure 3.
BTO rings during the MH370 flight [3]. The latitude of the arrow shown for each ring is arbitrary.

$$\Delta F_k^{\text{up}}\left(\mathbf{x}_k,\mathbf{s}_k\right)=\frac{v_z\cdot F^{\text{up}}}{c}, \qquad (5)$$

where F^{up} and c are as previously defined and v_z is the vertical speed of the plane. Substituting an uplink frequency of 1,646.6525 MHz (the uplink frequency stated in [1]) and a vertical velocity of 100 feet per minute (fpm), equivalent to 0.508 meters per second, (5) implies that the predicted BFO would increase by 2.8 Hz per 100 fpm of climb rate or decrease by 2.8 Hz per 100 fpm of descent rate if the plane were directly below the satellite. This is the maximum possible contribution of the plane's climb or descent rate to the BFO. In the more general case, (5) is moderated by the sine of the elevation angle θ from the aircraft to the satellite. This is expressed as

$$\Delta F_k^{\text{up}}\left(\mathbf{x}_k,\mathbf{s}_k\right)=\frac{v_z\cdot F^{\text{up}}\sin(\theta)}{c}. \qquad (6)$$
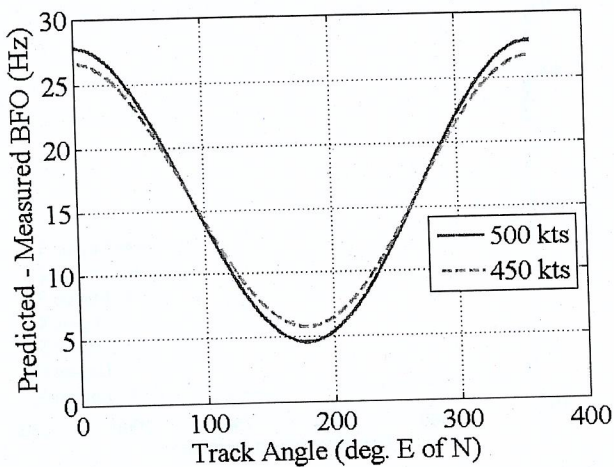
As such, at 00:19Z (the time at which the plane crosses the seventh arc, provided by Figure 3),[5] where the elevation to the satellite is 38.8°, the contribution to the BFO of climb or descent rate is reduced to approximately +1.7 or −1.7 Hz per 100 fpm, respectively.

EFFECT OF TRACK ANGLE ON BFO TOWARD END OF FLIGHT

Another factor that needs to be considered in interpreting the BFOs at 00:19Z is the track angle of the aircraft. In [2], Figure 5.6, a set of curves was shown, illustrating the relationship of the BFO error and the aircraft track angle at 18:39Z. That

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[5] The arcs referred to in this article and other MH370 literature are segments of the rings in Figure 3.

Figure 4.
BFO errors as a function of track angle at 00:11Z.

Figure 5.
Measured BFOs during the MH370 flight, with a trend line extrapolated back to 18:40Z and forward to 00:19Z.

was done to illustrate that under the assumption of level flight during the unanswered telephone call period of 18:39Z–18:41Z, MH370 would have been tracking in a southerly direction. The same model can be used to examine the relationship between the BFO error and the track angle at 00:11Z, when MH370 crossed the sixth arc, 8 min before the last messages were received from MH370.[6] To generate the curve of BFO error vs. track angle, it is assumed that the aircraft crossed this arc at 38.67° S, 85.11° E. A previous sensitivity analysis has revealed the curve is relatively insensitive to the actual crossing point. In addition, calculations revealed that (as shown in Figure 5.6 of [2] at 18:40Z) the track angle of the aircraft influences the BFO to a greater degree for a faster assumed ground speed. To assess the maximum possible effect of track angle on the BFO at 00:19Z, a ground speed of 500 knots is assumed, and the BFO error vs. track angle is then as shown in Figure 4. Another curve assuming 450 knots of ground speed is shown for comparative purposes. The peak-to-peak variations of the BFO difference are similar for both considered ground speeds.

BFO TREND DURING THE MH370 FLIGHT

The measured BFOs from 19:41Z to 00:11Z are shown in Figure 5, together with a line of best fit. This line is extended forward to the 00:19Z log-on, resulting in an expected BFO of roughly 254 Hz.[7] With reference to the track angle curves presented in Figure 4, the 00:11Z BFO error value for the southernmost track is roughly 6 Hz, meaning the measured BFO of 252 Hz was 6 Hz lower than the lowest value it could have been (assuming roughly level flight at approximately 450 knots). If we assume the same error value at 00:19Z and use the extrapolated BFO, assuming a south track with level flight and similar ground speed, the expected BFO value is 260 Hz.

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[6] The sixth arc crossing is considered here to isolate effects of ground track angle variation as opposed to vertical velocity, because the measured BFO at the sixth arc is still consistent with level flight.

[7] BFOs observed at 00:19Z were much lower than the expected value.
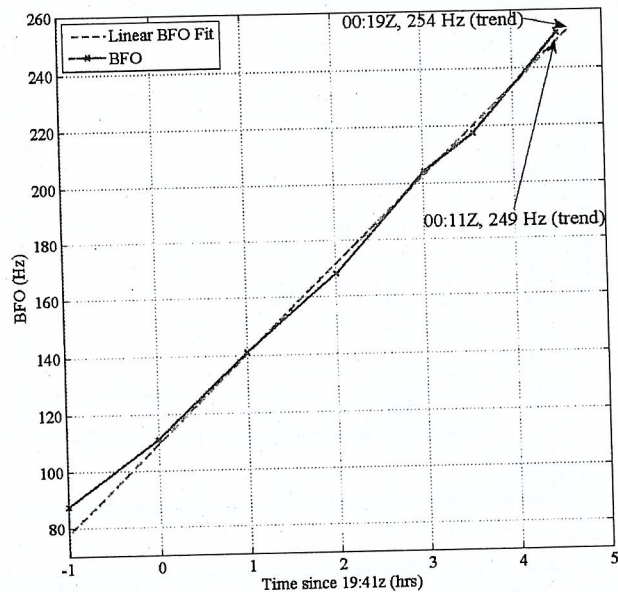
The linear trend line in Figure 5 is also extended back by 1 h to the time of the first unanswered satellite telephone call to MH370. The mean of the measured BFOs during that call attempt is also shown. Because the mean of the measured BFOs from the 18:39Z–18:41Z call attempt is in broad agreement with the linear trend observed in the BFOs from 19:41Z to 00:11Z (for which the BTOs were consistent with straight and level flight [2]), this supports the finding in [2] that there were most likely no major turns after the unanswered call attempt (see [2], Figure 10.5).

EFFECT OF SDU START-UP ON THE BFO

The BFOs logged for messages from the plane received at the Perth GES at 00:19:29Z and 00:19:37Z, 8 March 2014, were much lower than the expected value of 260 Hz. In the absence of other factors affecting the BFOs at these times, this suggests a large uncompensated negative vertical velocity component in the BFOs. In determining how to interpret this data, it is important to carefully consider other known factors that could affect these BFOs. One potential factor is an SDU oscillator start-up transient. This section analyses the range of expected effects on the BFO caused by a period of power outage followed by a restart of the SDU, as was a possibility between 00:11Z and 00:19Z on 8 March 2014 (see [3] for further details).

DETAILED ANALYSIS OF SDU START-UP EFFECTS ON THE BFO

At 18:25:27Z, 7 March 2014, the Inmarsat GES in Perth received a SATCOM log-on request from 9M-MRO. A series of messages was exchanged in the following few minutes as part of a standard log-on sequence. The BFOs over those minutes displayed somewhat unusual behavior in that (barring the first BFO) the BFO appeared to be exhibiting a transient settling behavior. This is shown in Figure 6. It was noted in [1] that the spike in BFO observed at
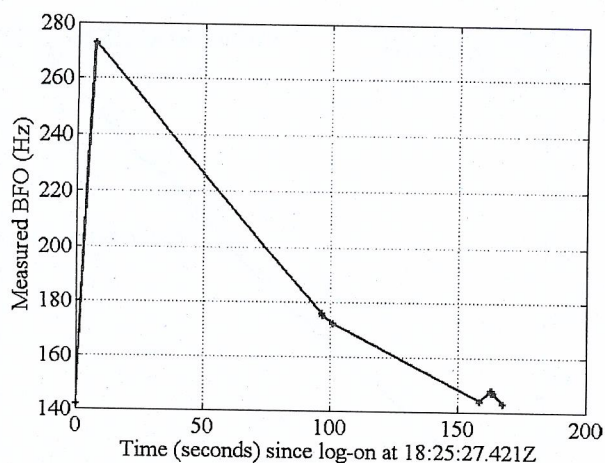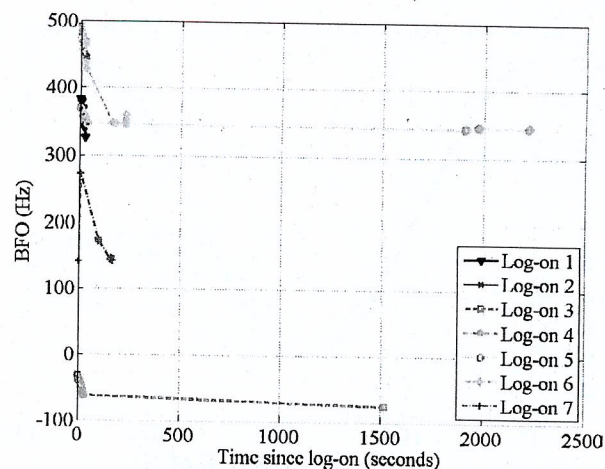
Figure 6.
Measured BFOs following the 18:25:27Z log-on message from MH370.

Figure 7.
Measured BFOs for seven log-ons of 9M-MRO.

this time[8] was not fully understood, and while originally attributed to a possible aircraft maneuver, it could be related to frequency changes occurring during the log-on sequence. A subsequent study [8] by the SATCOM subgroup of the MH370 Flight Path Reconstruction Group revealed this was most likely due to power-on frequency drift and subsequent stabilization of the oven-controlled crystal oscillator (OCXO) in the SDU. In [8], several SDUs were tested to investigate the effects of power outages on the BFO during SDU power-on, triggering a SATCOM log-on, such as occurred for MH370 at 18:25:27Z. It was found that while the frequency settling behavior was different for each SDU, any given SDU displayed repeatable behavior for a fixed outage duration and similar settling characteristics for outages of different duration.

Based on sequences of Inmarsat BFO logs for 9M-MRO concerning four log-ons after periods of SATCOM outage in the week leading up to the accident flight, it was found (in [8]) that the SDU in 9M-MRO resulted in a BFO that was initially too high at the time of log-on, followed by a simple decay over the next few minutes to a steady-state value. Regarding the log-on event at 18:25:27Z for MH370, there was a nonzero bit error rate (BER) associated with the log-on request at that time. The associated received signal level and carrier-to-noise density ratio (C/N_0) were also unusually low. As such, the first BFO was deemed untrustworthy.[9] This is not the only time such a behavior was observed for the SDU in 9M-MRO. In the analysis of BFOs presented in [2], there was an outlier in the BFO (with an error of −170 Hz) found to have occurred for a SATCOM message associated with 9M-MRO in 1 of the 20 previous flights of 9M-MRO. This was discarded as an outlier and not presented in [2]. It too had a nonzero BER and a low C/N_0 (37.6 dBHz compared to surrounding values between 41.5 and 42 dBHz). With the removal of the untrustworthy BFO logged for the first message in the sequence

shown in Figure 6, the 18:25Z log-on from MH370 was determined to follow the simple decay trend observed in other instances.

It was noted by DST Group that if the simple BFO decay trend established after SDU power-up [8] also occurred during the MH370 SATCOM log-on event beginning at 00:19:29Z, then it would allow bounds to be established on the BFO that would have been expected for MH370 if the plane was flying level at that time. This in turn would allow the determination of bounds on the possible descent rates of MH370 during that final log-on event. To build confidence that the simple BFO decay trend would hold for the 00:19Z log-on event, DST Group reviewed additional Inmarsat logs for 9M-MRO, corresponding to the period 22 February to 28 February 2014. Two additional cases were identified in which 9M-MRO logged back onto an Inmarsat satellite after a sustained period of SATCOM outage. The sequence of BFOs observed in these two cases, along with the five considered by the SATCOM working group in [8], are shown in Figure 7, and details about each of the log-ons are given in Table 2.

The periods of SATCOM outages followed by log-on events were identified from Inmarsat-provided ground-station logs by identifying sequences of three or more unsuccessful log-on interrogations to 9M-MRO (suggesting the SDU was likely powered off), followed by a log-on to the satellite system initiated from 9M-MRO at some later time. The exact duration of the power-off period for the SDU was unable to be determined from the logs; however, the time stamps associated with the unsuccessful log-on interrogations and subsequent 9M-MRO initiated log-ons were used to determine bounds on the outage time, as shown in the third column of Table 2.[10] Additional comments about each log-on sequence are given in the final column. While some BERs for log-ons 1 and 6 were nonzero, the BFOs did not appear to be outliers and were therefore considered valid. The uncertainty window in the length of the outage durations shown is roughly 60 min; this relates to timers in the Inmarsat satellite ground stations that result

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[8] When viewed on a timescale of hours, as shown in Figure 9 of [1], this behavior looks like a spike in BFO between 18:25Z and 18:28Z.

[9] An alternative explanation for this outlier BFO could be the unknown conditions of the aircraft between 17:22Z and 18:25Z; for instance, the temperature of the SDU might have been much lower than the expected ambient temperature.

[10] The bounds shown are derived under the assumption that the SATCOM outage was exclusively due to a power outage. It is also possible that the first part of some outages was due to a blocked line of sight to the Inmarsat satellites.

Table 2.

Details of Log-on Sequences Used for Analysis

Identifier	Date and Time Stamp of Log-on	Duration of Preceding Power Outage	Other Comments
Log-on 1	23 Feb. 2014 23:57Z	Between 381 and 442 min	After scheduled A1 maintenance check, some nonzero BERs
Log-on 2	26 Feb. 2014 14:11Z	Between 295 and 354 min	—
Log-on 3	5 Mar. 2014 03:06Z	Between 35 and 95 min	—
Log-on 4	6 Mar. 2014 13:29Z	Between 43 and 103 min	—
Log-on 5	6 Mar. 2014 15:02Z	Between 35 and 92 min	—
Log-on 6	7 Mar. 2014 12:50Z	Between 228 and 288 min	Some nonzero BERs
Log-on 7	7 Mar. 2014 18:25Z	Between 20 and 78 min	First point untrustworthy

in a log-on interrogation being sent to the plane after roughly 60 min of SATCOM inactivity. It is this same timing responsible for the roughly 60-min separation between the four BTO arcs from 19:41Z to 22:41Z (e.g., [1] and [3]). In addition, based on other evidence related to MH370 (e.g., secondary radar transponder loss of signal at 17:22Z), SDU power was probably first lost at 17:22Z, in which case the SDU power outage duration for log-on 7 would be approximately 63 min.
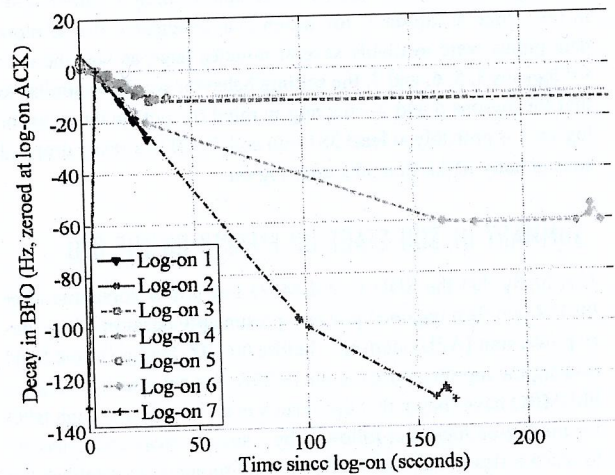
For log-ons 3 and 5, there were some points available about half an hour after the log-on in each case (noting the plane was on the tarmac the whole time) that demonstrate the BFO appears to have fully settled to a steady-state value. Log-ons 1 to 5 lasted less than 1 min after the log-on request message. The results for log-ons 6 and 7, which had log-on sequences lasting a few minutes, suggest a settling behavior of approximately 3 min. For all observed log-on sequences, there seems to be a simple decay of BFO over a few minutes after SDU start-up and log-on request (Figure 8). It is therefore possible a similar decay was occurring during the SDU log-on sequence beginning at 00:19:29Z.

To establish what the likely range of steady-state equivalent BFOs would have been at 00:19Z, it is useful to replot the curves from Figure 7, translating them vertically such that the log-on acknowledgment has a 0-Hz offset. This is done in Figure 8, which has also been zoomed in for the time axis for clarity.

For log-ons 1 to 6, the aircraft was on the ground at an airport. It has been established that the most common scenario for an on-ground, at-airport log-on after a sustained power outage of the SDU is for the AES to use a different self-Doppler compensation method than was in use for log-on 7 and the log-on at 00:19Z. This is because of a lack of navigational input data from the aircraft's inertial navigation unit in the few minutes after power-up and is called the closed-loop Doppler compensation approach.[11]

The standard AES Doppler compensation mode when navigational data are available to the SDU is the mode used for log-on 7 and the log-on at 00:19Z. This approach was described previously.

Figure 8.
Decay in measured BFOs for seven log-ons of 9M-MRO. The decay rates for log-ons 1 to 6 have been adjusted to account for the different AES Doppler compensation mode used during these log-ons.

It used the aircraft's own estimates of its position and horizontal velocity to predict and correct the AES-contributed Doppler shift. This is called open-loop Doppler compensation.

For log-ons 1 to 6, we work on the premise that closed-loop Doppler compensation was used.[12] In that approach, the frequency of broadcast P-channel transmissions received by 9M-MRO were used to determine the AES Doppler contribution. In this approach, all of the difference between the received and the expected P-channel frequencies is attributed by the SDU to be caused by AES motion-induced Doppler. As such, if the OCXO frequency is higher than its nominal settled value, because of incomplete temperature stabilization, the received frequency will appear lower than expected by the corresponding amount. When applying the AES Doppler compensation in this case, the transmit frequency is increased by

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[11] The use of closed-loop Doppler compensation in this circumstance was established in consultation with members of the MH370 Flight Path Reconstruction Group, including representatives from Thales and Inmarsat.

[12] Previous analysis had not considered the difference in Doppler compensation mode yet resulted in descent rates similar to those presented in this paper.

the difference between the stable and the prestabilized oscillator frequency (scaled to account for the difference between P-channel frequency and the R-channel transmit frequency). Moreover, the transmit frequency (before AES Doppler compensation) will already be too high by the same amount because of the incomplete temperature stabilization. The net combined effect to the BFO decay observed due the OCXO settling will therefore be doubled compared to the case in which open-loop AES Doppler precompensation is used. Hence, the BFO decay results for log-ons 1 to 6 shown in Figure 8 have been halved for the analysis when compared to those inferred from the raw measured BFOs presented in Figure 7.

Recall that the two messages at 00:19:29Z and 00:19:37Z are a log-on and a log-on acknowledgment, respectively. From the data used to plot Figure 8, it can be determined that the log-on acknowledgment BFOs are in the range of [0, 6] Hz lower than the log-on BFOs. In addition, the maximum difference between the log-on BFO and the settled BFO value is in the range [17, 136] Hz. The only log-on that doesn't appear to be approaching a settled value in this range is log-on 1, for which it is anticipated that if more data points were available several minutes later, as was the case for log-ons 3, 5, 6, and 7, the settling behavior would be similar to that for log-ons 6 and 7—bearing in mind the outage duration for log-on 1 is probably at least 381 min and the BFO already dropped substantially in the first 30 s after log-on.

SUMMARY OF SDU START-UP EFFECTS ON THE BFO

It is likely that the SDU in 9M-MRO lost power sometime after 00:11Z and then regained power—presumably because of auxiliary power unit (APU) start-up—before 00:19Z, leading to the SDU making the log-on attempt at 00:19:29Z. Previous such events for 9M-MRO have shown that this results in an initially too-high BFO for the log-on message, followed by a simple decay characteristic to reach a steady-state BFO after several minutes. To establish the relevance of the BFOs at 00:19:29Z and 00:19:37Z to the motion (including descent rate) of MH370 at those times, this section has established bounds for the likely range of steady-state equivalent BFOs at those times. This was done by analyzing BFOs from seven previous SDU start-up events for a period of several minutes. The specific results obtained suggest the following:

1. The recorded BFO for the 00:19:29Z log-on was between 17 and 136 Hz higher than it would have been if the OCXO in the SDU was in a steady state at that time.

2. The recorded BFO for the 00:19:37Z log-on acknowledgment message was between 17 and 130 Hz higher than it would have been if the OCXO in the SDU was in a steady state at that time.

BOUNDING THE DESCENT RATES OF MH370

The results from the previous two sections can be combined to provide bounds on the descent rate of MH370 implied by the BFOs from the two last SATCOM messages for the flight. In this section, it is shown how this is done for two possibilities that could explain the attempted SATCOM log-on from 9M-MRO at 00:19Z. In [3], the most likely cause of this log-on was stated to be a power

Table 3.

The Last Two BFOs for MH370 Under Hypothesis I			
Time Stamp	Recorded BFO (Hz)	BFO Range If Start-Up Drift Is Removed (Hz)	Extended Range Considering BFO Noise (Hz)
00:19:29Z	182	[46, 165]	[28, 193]
00:19:37Z	−2	[−132, −19]	[−150, 9]

interruption resulting from insufficient fuel and subsequent engine flameout. It could also have been because of a temporary software failure, a loss of systems providing critical input to the SDU, or a loss of the SATCOM link because aircraft attitude was such that the line of sight to the satellite is blocked. If it was a power interruption to the SDU caused by loss of fuel and subsequent reboot using the APU, the SDU would be without power for about 1 min. In this case, the summary of SDU start-up effects on the BFO discussed in a previous section need to be considered when interpreting the last two BFOs.[13] However, if the power loss was momentary (resulting in a reset of the SDU) or if the temporary SATCOM outage leading to the log-on request was due to one of the other listed reasons, there would be no warm-up drift to consider, so the effect of SDU start-up on the BFO would not need to be applied. Both cases are considered separately in the following two subsections, and then combined overall bounds are presented.

HYPOTHESIS I: SATCOM OUTAGE DUE TO INSUFFICIENT FUEL

If the SDU log-on at 00:19:29Z was due to engine flameout, followed by a restart of the SDU using power from the APU, the SDU outage preceding the log-on would have lasted about 1 min. This would result in some cooling of the OCXO in the SDU. The likely effect of the SDU start-up and consequent warm-up drift of the OCXO is summarized in the previous section. Table 3 presents the recorded BFOs and bounds on the adjusted BFOs to remove these effects of warm-up drift. In the last column of the table, the bounds are extended, taking into account the BFO noise bounds of [−28, +18] Hz established in the section that reviews the BFO statistics.

As established in the previous section on the effects of aircraft position and velocity on the BFO, the recorded BFO would be roughly 1.7 Hz lower for every 100 fpm of descent rate. As such, depending on whether the plane was tracking south or north (minimum or maximum expected BFOs, respectively), bounds on the descent rate of MH370 at the times of transmission corresponding to the last two BFOs can be determined by subtracting the values given in the rightmost column of Table 3 from the expected BFO values for level flight tracking south or north, dividing the result

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[13] Given the relatively short duration of the power-loss under Hypothesis 1, it is possible that the extent of BFO decay due to OCXO warm-up would be less than shown for log-ons 1 to 7. This is essentially covered within Hypothesis 2, which considers a momentary SATCOM outage.

Table 4.

MH370 Descent Rates at 00:19Z, 8 March 2014, Under Hypothesis 1				
Time Stamp	Min. Desc. Rate, South Track (fpm)	Min. Desc. Rate, North Track (fpm)	Max. Desc. Rate, South Track (fpm)	Max. Desc. Rate, North Track (fpm)
00:19:29Z	3,900	5,100	13,600	14,800
00:19:37Z	14,800	15,900	24,100	25,300

Min., minimum; **Max.**, maximum; **Desc.**, descent.

Table 5.

The Last Two BFOs for MH370 Under Hypothesis 2		
Time Stamp	Recorded BFO (Hz)	BFO Bounds with Noise (Hz)
00:19:29Z	182	[164, 210]
00:19:37Z	-2	[-20, 26]

by 1.7 Hz and multiplying by 100 fpm. The expected BFO for a south track is approximately 260 Hz, while for a north track it is close to 280 Hz. Using these numbers, it is straightforward to obtain the bounds shown in Table 4. The bounds have been rounded to the nearest 100 fpm. Looking at all values in the table, it can be concluded that irrespective of ground track angle and for assumed ground speeds less than approximately 500 knots, under Hypothesis 1, MH370 would have been descending between 3,900 and 14,800 fpm at 00:19:29Z and 8 s later would have been descending between 14,800 and 25,300 fpm. These descent rates are consistent with simulations of an uncontrolled phugoid descent reported in [6].

HYPOTHESIS 2: SATCOM OUTAGE DUE TO SOME OTHER REASON

It is still of interest to determine bounds on the descent rates under the alternate hypothesis that something not associated with a power outage led to the SDU-initiated log-on event at 00:19Z. In this case, the warm-up drift would not apply. Therefore, the recorded BFOs can be treated normally (though still subject to BFO noise). In this case, the BFO bounds are as set out in Table 5. Using these bounds, the lower and upper descent rates can be calculated again, resulting in the bounds presented in Table 6. It can be concluded from the table that the recorded BFOs indicate that at 00:19:29Z,

the plane was descending at a rate between 2,900 and 6,800 fpm. At the time of the last SATCOM message, it can be concluded under Hypothesis 2 that the descent rate would have been between 13,800 and 17,600 fpm.

SUMMARY OF DESCENT RATE BOUNDS

Combining the descent rate bounds for Hypotheses 1 and 2, outer bounds can be established for the descent rates of MH370 at 00:19:29Z and 00:19:37Z. Specifically, regardless of ground track angle and for ground speeds less than approximately 500 knots, accounting for possible BFO noise and regardless of whether the SATCOM outage between 00:11Z and 00:19Z was due to an SDU power outage or another reason, the outer bounds on the possible descent rates at the times of the last two SATCOM messages from MH370 are given in Table 7.

ESTIMATED DOWNWARD ACCELERATION

When interpreting the bounds presented in Table 7, the conditions under which each bound was derived need to be considered. For instance, the lowest rate at 00:19:29Z was derived assuming no period of SDU outage and a southward track, whereas the highest descent rate for 00:19:37Z was derived assuming a 1-min outage of the SDU with the maximum BFO decay observed in the previous seven SDU outage events for 9M-MRO. Hence, it would not be reasonable to say that the plane could have been descending at 2,900 fpm at 00:19:29Z and 25,300 fpm 8 s later, which would imply a downward acceleration on the order of 2,800 fpm per second. While it is not possible to determine a precise acceleration value, a rough approximation of the average descent rate over those 8 s could, for instance, be taken using the midpoints of the bounds at each time, which would result in an average downward acceleration of 10,700 fpm in 8 s, or around

Table 6.

MH370 Descent Rates at 00:19Z, 8 March 2014, Under Hypothesis 2				
Time Stamp	Min. Desc. Rate, South Track (fpm)	Min. Desc. Rate, North Track (fpm)	Max. Desc. Rate, South Track (fpm)	Max. Desc. Rate, North Track (fpm)
00:19:29Z	2,900	4,100	5,600	6,800
00:19:37Z	13,800	14,900	16,500	17,600

Min., minimum; **Max.**, maximum; **Desc.**, descent.

Table 7.

Range of Possible MH370 Descent Rates at 00:19Z, 8 March 2014		
Time Stamp	**Min. Desc. Rate (fpm)**	**Max. Desc. Rate (fpm)**
00:19:29Z	2,900	14,800
00:19:37Z	13,800	25,300

Min., minimum; **Max.,** maximum; **Desc.,** descent.

1,300 fpm per second. This equates to around 6.7 ms^{-2} or 0.68g, where g denotes Earth's gravitational constant, which is approximately 9.8 ms^{-2}. It is straightforward to see that other reasonable methods of estimating the downward acceleration (such as taking the difference between the two minimum descent rates or the two maximum descent rates at 00:19:29Z and 00:19:37Z) would yield similar acceleration values.

CONCLUSION

This article discussed all known factors that could have influenced the BFOs associated with the last two SATCOM messages received from MH370 at 00:19Z. Lower and upper bounds on its descent rate at this time were then derived.[14] The downward acceleration over the 8-s interval between these two messages was found to be approximately 0.68g. The derived bounds and approximate downward acceleration rate are consistent with simulations of an uncontrolled descent near the seventh arc, as reported in [6]. This suggests that 9M-MRO should lie relatively close to the seventh BTO arc.

At the time of the submission of this paper, MH370 had not been located. The search was in indefinite suspension pending "credible new evidence pointing to a specific location for the plane." Further efforts in data analysis and in drift modeling of located MH370 debris have been conducted and reported (e.g., [6], [9], and [10]). The descent analysis from this paper was used in [6] when considering the width of any potential future search area. Some recent news reports have suggested a U.S. underwater exploration firm named Ocean Infinity may soon be launching a renewed search effort for MH370. Refer to the ATSB website [11] and the general media for updates. ◆

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[14] The conclusions regarding descent rate have been made taking into account BFO noise bounds, all reasonably feasible ground tracks and speeds, and possible OCXO warm-up drift. It has been implicitly assumed that there were no otherwise-unknown factors that could have affected the last two BFOs.

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